ADA PINPOINT TOPIC PACKS

(1)Proof (13 Qns)
(2)Algebraic Proofs (1 Qns)

30_to_100_Percent_Pinpoint_AI_Pack

Time Allocation = 45mins , Max = 40 Marks

Calculated Grade Boundaries:

Grade	Marks
3+	3
4-	5
4	7
4+	9
5-	11
5	13
5+	15
6-	17
6	19
6+	22
7-	24
7	26
7+	28
8-	30
8	32
8+	34
9-	36
9	38
9+	40

Question 1 (AO2): 41% of students got this right (3 marks)

18. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(Total 3 marks)

Question 2 (AO2): (No Calc) 29% of students got this right (3 marks)

19. Prove algebraically that the sum of the squares of two consecutive integers is always an odd number.

(Total 3 marks)

Question 3 (AO2): (No Calc) 28% of students got this right (4 marks)

15. n is an integer greater than 1.

Use algebra to show that $(n^2 - 1) + (n - 1)^2$ is always equal to an even number.

(Total 4 marks)

Question 4 (AO2): (No Calc) 27% of students got this right (4 marks)

20. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total 4 marks)

Question 5 (AO2): 24% of students got this right (3 marks)

13 For any three consecutive whole numbers, prove algebraically that the largest number and the smallest number are factors of the number that is one less than the square of the middle number.

Question 6 (AO2): 21% of students got this right (3 marks)

16. Prove algebraically that the product of two odd numbers is **always** an odd number.

Question 7 (AO2): 15% of students got this right (3 marks)

20. Proof that $(n + 5)^2 - (n - 5)^2$ is even if n is a positive integer.

(Total 3 marks)

Question 8 (AO2): 15% of students got this right (3 marks)

17 The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

Question 9 (AO1): (No Calc) 14% of students got this right (4 marks)

16 n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(Total for Question 16 is 4 marks)

Question 10 (AO1): (No Calc) 13% of students got this right (3 marks)

19. Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12.

(Total for Question 19 is 3 marks)

Question 11 (AO1): (No Calc) 11% of students got this right (3 marks)

13 Prove algebraically that the sum of any two different odd numbers is an even number.

(Total for Question 13 is 3 marks)

Question 12 (AO2): 5% of students got this right (4 marks)

17. a, b, c are positive integers such that $a > b > c$.

N is the largest three digit number that has the digits a, b and c.
K is the smallest three digit number that has the digits a, b and c.

(a) Use algebra to show that the difference between N and K is always a multiple of 99.

(3)

(b) If $a > b$ and $b = c$ will the difference between N and K still be a multiple of 99?
Justify your answer.

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(1)

(Total for Question 17 is 4 marks)

Answers to Qn 1 (AO2): 41% of students got this right

	18.	$4n^2 + 12n + 3^2 - (4n^2 - 12n + 3^2)$ $= 4n^2 + 12n + 9 - 4n^2 + 12n - 9$ $= 24n$ $= 8 \times 3n$	Proof	3	M1 for 3 out of 4 terms correct in expansion of either $(2n + 3)^2$ or $(2n - 3)^2$ A1 for $24n$ from correct expansion of both brackets A1 (dep on A1) for $24n$ is a multiple of 8 or $24n = 8 \times 3n$ or $24n \div 8 = 3n$

Answers to Qn 2 (AO2): (No Calc) 29% of students got this right

Question		Working	Answer	Mark	Notes
19.		$x^2 + (x + 1)^2$ $= x^2 + x^2 + 2x + 1$ $= 2x^2 + 2x + 1$ = even + even + odd = odd	proof	3	M1 for $x^2 + (x + 1)^2$ or $(x - 1)^2 + x^2$ oe M1 for correctly expanding $(x + 1)^2$or$(x - 1)^2$ C1 for simplifying correctly and for final explanation and states x is an integer, e.g. $2(x^2 + x)$ is even and 1 is odd **and e**ven + odd is odd

Answers to Qn 3 (AO2): (No Calc) 28% of students got this right

15.			Correct proof	4	M1 expands $(n-1)^2$ with at least three out of four terms correct **or** $n^2 - n - n + 1$ **or** $n^2 - 2n + 1$ M1 $n^2 - 1 + n^2 - n - n + 1$ **or** $2n^2 - 2n$ A1 $2(n^2 - n)$ **or** $2n(n-1)$ C1 (dep on M1) for conclusion $2 \times$ '$(n^2 - n)$' or $2 \times n \times$ '$(n-1)$' is always even OR M1 factorises $n^2 - 1$ correctly $(n-1)(n+1)$ M1 $(n-1)(n+1+n-1)$ A1 $2n(n-1)$ C1 (dep on M1) for conclusion $2 \times$ '$(n^2 - n)$' or $2 \times n \times$ '$(n-1)$' is always even

Answers to Qn 4 (AO2): (No Calc) 27% of students got this right

20.		$(n+1)^2 - n^2$ $= n^2 + 2n + 1 - n^2$ $= 2n + 1$ $(n+1) + n = 2n + 1$ **OR** $(n+1)^2 - n^2$ $= (n + 1 + n)(n + 1 - n)$ $= (2n + 1)(1) = 2n + 1$ $(n+1) + n = 2n + 1$ **OR** $n^2 - (n+1)^2 =$ $n^2 - (n^2 + 2n + 1) =$ $-2n - 1 = -(2n + 1)$ Difference is $2n + 1$ $(n+1) + n = 2n + 1$	proof	4	M1 for any two consecutive integers expressed algebraically e.g. n and $n+1$ M1 (dep on M1) for the difference between the squares of 'two consecutive integers' expressed algebraically e.g. $(n+1)^2 - n^2$ A1 for correct expansion and simplification of difference of squares, e.g. $2n + 1$ C1 (dep on M2A1) for showing statement is correct, e.g. $n + n + 1 = 2n + 1$ and $(n+1)^2 - n^2 = 2n + 1$ from correct supporting algebra

Answers to Qn 5 (AO2): 24% of students got this right

Question	Working	Answer	Mark	Notes
13		Proof	M1	for 3 consecutive integers written algebraically, e.g. n, $n + 1$, $n + 2$ **or** $n - 1$, n, $n + 1$
			M1	for multiplying the smallest and largest, e.g. $n(n + 2) = n^2 + 2n$ **or** $(n - 1)(n + 1) = n^2 - 1$ **or** for squaring the middle number
			C1	for a correct conclusion from correct expressions

Answers to Qn 6 (AO2): 21% of students got this right

Question		Working	Answer	Mark	Notes
*16		$(2n + 1)(2m + 1)$ $= 4nm + 2n + 2m + 1$ $= 2(2nm + n + m) + 1$	Proof	3	M1 for $2n + 1$ oe used to describe an odd number A1 for product = $4nm + 2n + 2m + 1$ where n is not the same as m C1 (dep on M1) for stating that $2 \times$ '$(2nm + n + m)$' is even since it is a multiple of 2 so adding 1 gives an odd number

Answers to Qn 7 (AO2): 15% of students got this right

20 Show that $(n + 3)^2 - (n - 3)^2$ is an even number for all positive integer values of n.

$(n^2 + 6n + 9) - (n^2 - 6n + 9)$

$= 12n$

$= 6 \times 2n$

$\therefore$ even for all n.

Answers to Qn 8 (AO2): 15% of students got this right

Paper 1MA1: 3H			
Question	**Working**	**Answer**	**Notes**
17		proof	C1 starts proof eg $n(n+1)$ or $(n-1)\times n$ C1 $n(n+1) + n+1$ or $(n-1)\times n + n$ C1 for convincing proof including $(n+1)^2$ or n^2

Answers to Qn 9 (AO1): (No Calc) 14% of students got this right

Question 16 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(n-2)^2 = n^2 - 4n + 4$	C1	This mark is given for a correct expansion of $(n-2)^2$
	$n^2 - 2 - n^2 + 4n - 4$	C1	This mark is given for a correct expansion of $n - 2 - (n-2)^2$
	$2(2n - 3)$	C1	This mark is given for reducing the expression to $(2n - 3)$
	$2(2n-3)$ always even since it has a factor of 2 for all values of n	C1	This mark is given for a correct conclusion supported by working shown

Answers to Qn 10 (AO1): (No Calc) 13% of students got this right

19		shows result	C1	shows expansion of the squares of any three consecutive numbers shown algebraically, e.g. $(4n^2 + 4n + 1)$ or $(4n^2 + 12n + 9)$ or $(4n^2 + 20n + 25)$
			C1	simplifies , e.g. $12n^2 + 36n + 35$
			C1	arrives at $12(n^2 + 3n + 2) + 11$ (oe) and concludes result

Answers to Qn 11 (AO1): (No Calc) 11% of students got this right

13 Prove algebraically that the sum of any two different odd numbers is an even number.

$$(2n + 1) + (2m + 1)$$
$$= 2n + 2m + 2$$
$$= 2(n + m + 1)$$

So sum is even number

(Total for Question 13 is 3 marks)

Answers to Qn 12 (AO2): 5% of students got this right

17	(a)		$99(a - c)$	M1	for forming $100a + 10b + c$ or $100c + 10b + a$
			with	M1	for finding the difference for their expressions
			conclusion	C1	for a concluding statement with $99(a - c)$
	(b)		statement	C1	e.g. has no effect , b's cancel